**UNIVERSITY OF TORONTO  
Faculty of Arts and Science**

**DECEMBER 2016 EXAMINATIONS**

**PHL245H1-F**

**Alex Koo**

**Duration - 3 hours**

**No Aids Allowed**

Last Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

First Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Student Number: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Answer **ALL** questions on the exam paper.

Use examination booklets for rough work if needed.

If you need further space, use an examination booklet and clearly indicate on the exam paper where your solution is.

The exam consists of 16 pages. Pages 2-15 have questions on them.

The final page (16)is a blank lined page for use if needed.

Part I: Semantics (30 marks)

1. Explain why an argument with a contradiction in the premises is valid. What does this tell us about the concept of validity? (4)

2. Provide an intensional interpretation that shows the following set of sentences is consistent. (3)

{Ga∧~Fa, ∃y∀x(Fx→(Gy∧L(yx)), ∃z(Fz∧L(za))}

3. Provide an English explanation that demonstrates the following sentence is a logical falsehood/contradiction. (4)

∃x(Fx∧∀y~G(yx))∧∀xG(xx)

4. Provide a finite extensional interpretation/model that demonstrates the following argument is invalid. (4)

∃x(Fx∧Gx∧~D(xx)). ∀x(Gx→∃y(Fy∧D(xy)). ∃x(Hx∧~(Fx∨Gx)).   
∴ ~∀x(Hx→∀y(Fy→D(xy)))

5. a) Provide a truth-functional expansion of the following argument using a  
 universe of discourse with two members. (4)

∀x(Fx→∃yB(yx)). ~∀z(Gz∧B(zz)). ∴ ∃y∀x(B(xy)∧Gx).

b) Provide a finite extensional interpretation/model that demonstrates the argument from part (a) is invalid. (1)

6. We know that a disjunction can actually be expressed as a conditional of the form “if not one, then the other.” Given this, it is clear that we could easily remove the disjunction entirely from our logical system without impacting its completeness. Briefly give some reasons why we should NOT remove the disjunction from our logical system. (2)

7. Below is the truth table for the logical connective called the Sheffer Stroke symbolized by the vertical bar, ∣. Convert each of the following sentences into a logically equivalent sentence that contains ONLY the Sheffer Stroke as its logical connectives.

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| P | Q | P ∣ Q |
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | T |

a) ~P (1)

b) P∧Q (2)

8. **Circle** the single best answer to the following LSAT logic question: (2)

Several critics have claimed that any contemporary poet who writes formal poetry—poetry that is rhymed and metered—is performing a politically conservative act. This is plainly false. Consider Molly Peacock and Marilyn Hacker, two contemporary poets whose poetry is almost exclusively formal and yet who are themselves politically progressive feminists.

The conclusion drawn above follows logically if which one of the following is assumed?

1. No one who is a feminist is also politically conservative.
2. No poet who writes unrhymed or unmetered poetry is politically conservative.
3. No one who is politically progressive is capable of performing a politically conservative act.
4. Anyone who sometimes writes poetry that is not politically conservative never writes poetry that is politically conservative.
5. The content of a poet’s work, not the work’s form, is the most decisive factor in determining what political consequences, if any, the work will have.

9. Provide a shortened truth-table that demonstrates the following set of sentences is not inconsistent. (3)

{P→~(R∧S), ~(R→(P↔Q)), ~(Q∨W)}

Part II: Symbolization (34 Marks)

Symbolize questions 1-8, and translate question 9 using the provided abbreviation schemes.

1. A sufficient condition for people and hamsters to feel good is exercising. (3)

E1: *a* exercises. F1: *a* is a person. G1: *a* feels good. H1: *a* is a hamster.

2. Although neither Demar nor Demar’s wife ever watch hockey games, they both know how to skate. (4)

d0: Demar. d1: The wife of *a*. A1: *a* is a time. D1: *a* knows how to skate.   
H1: *a* is hockey game. G3: *a* watches *b* at time *c*.

3. A person who doesn’t like cats is wise, and only in that case does he/she lead a successful life. (4)

A1: *a* leads a successful life. C1: *a* is a cat. D1: *a* is wise. F1: *a* is a person. L2: *a* likes *b*.

4. Unless they aren’t funded by the Government of Canada, parks that people visit are clean. (4)

c0: Canada. a1: The Government of *a*. C1: *a* is clean. F1: *a* is a person. G1: *a* is a park.   
A2: *a* visits *b*. F2: *a* funds *b*.

5. Despite the fact that Avery’s best friend is the silliest student at UofT, he doesn’t have any other friends (who are people) besides Avery. (4)

a0: Avery. b0: UofT. b1: The best friend of *a*. F1: *a* is a person. A2: *a* is a student at *b*. C2: *a* is sillier than *b*. F2: *a* is the friend of *b*.

6. No board games except for Dominion, which you need at least two people playing for it to be good, are fun. (4)

d0: Dominion. A1: *a* is fun. B1: *a* is a board game. F1: *a* is a person. G1: *a* is good.   
C2: *a* plays *b*.

7. Exactly one student who takes PHL245 will go on and ace third year logic. (4)

a0: PHL245. b0: Third year logic. A1: *a* is a student. A2: *a* takes *b*. B2: *a* will ace *b*.

8. Symbolize the following ambiguous sentence in TWO logically distinct ways. Provide an English sentence that clarifies the meaning of each symbolization. (4)

Some person doesn’t ever ride a bike.

B1: *a* is a bike. D1: *a* is a time. F1: *a* is a person. N3: *a* rides *b* at *c*.

9. Translate the following symbolic sentence into an IDIOMATIC English sentence using the provided abbreviation scheme. (3)

b(a)=a(b)∧∀x(Fx∧H(xc(a))→∀y(Fy∧H(yc(a))→x=y))

a0: Lois Lane. b0: Superman. a1: The alter ego of *a*.   
b1: The partner of *a*. c1: The secret of *a*. F1: *a* is a person. H2: *a* knows *b*.

Part III: Derivations (36 marks)

1. Show the following statement is a theorem of logic using a derivation. Use only the **basic** rules: MP, MT, ADD, MTP, ADJ, S, R, DN, CB, BC, EI, EG, and UI. (6)

∴ ∀x∃y(F(xy)∧Gx)→∃x∃z(F(a(x)z)∧Gz).

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2. Show the following argument is valid using a derivation. Use only the **basic** rules: MP, MT, ADD, MTP, ADJ, S, R, DN, CB, BC, EI, EG, and UI. (9)

∀x(Fx∧∀yH(yx)). ~∃yGy∨∃xBx. ∀z∃xH(a(z)x)→~∃z(Bz∨Az). ∴ ~∃x(Fx↔Gx).

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3. Show the following argument is valid using a derivation. Use only the **basic** rules: MP, MT, ADD, MTP, ADJ, S, R, DN, CB, BC, EI, EG, and UI. In addition, add the following rule to your derivation system: (6)

Where φβ is a substitution of **any** occurrences of α in φ

**Leibniz’s Law (LL)**

φα

α=β

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∴ φβ

∀x(Fx→x=a∨x=b). ∼Gb. ∃x(Fx∧Gx) ∴ Fa.

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4. Show the following argument is valid using a derivation. You may use the basic rules as well as the **derived** rules: CDJ, DM, NC, NB, SC, QN, and AV. (6)

∀x∀y∀z(L(xy)∧L(zy)→L(zx)). ∴ ~L(ab)→∀x~∀yL(yx).

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5. Show the following argument is valid using a derivation. You may use the basic rules as well as the **derived** rules: CDJ, DM, NC, NB, SC, QN, and AV. (9)

∀w~(∃zM(wb(z))↔∃yD(ya(yy))). ∀zFz∨∃x∀yD(a(x)y). ∴ ∀x(Fx∨~∀wM(b(x)w)).

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Total = 100 Marks

Extra Lines. If you use these, clearly indicate how the grader should read your proof.

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Total Pages (16)